Yang-Mills Fields Quantization in the Factor Space

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Abstract

The perturbation theory over inverse interaction constant $1/g$ is constructed for Yang-Mills theory. It is shown that the new perturbation theory is free from the gauge ghosts and Gribov's ambiguities, each order over $1/g$ presents the gauge-invariant quantity. It is remarkable that offered perturbation theory did not contain divergences, at least in the vector fields sector, and no renormalization procedure is necessary for it.

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1 Introduction

The perturbation theory for (3+1)-dimensional Yang-Mills field theory in vicinity of the extremum $\mathrm{u}_\mu^a(x)$ of the action will be described [1]. It is our first publication in this field and it seems reasonable to define from the very beginning the level of its completeness. Namely, we would like to show that, in opposite to the ordinary perturbative QCD (pQCD), the offered theory may be used at arbitrary distances. Accordingly, the theory is free from divergences at least in the vector fields sector. Besides the perturbation theory is operate with transparently gauge invariant quantities and no ghosts and Gribov ambiguities would hinder the computations.

We will realize the perturbation theory in the factor space $\mathcal{G}/\mathcal{H}$, where $\mathcal{G}$ is the symmetry group of theory and $\mathcal{H}$ is the symmetry of $\mathrm{u}_\mu^a(x)$. Introductory notes for this formalism was given in [2].

The usefulness of such choice follows from homogeneity and isotropy of $\mathcal{G}/\mathcal{H}$ in the semi-classical approximation. The developed perturbation theory is formulated for description of the violating these property quantum excitations. One may note that we offer the realization of perturbation theory in terms of the action-angle type variables. As an example one may have in mind the factor space [3]

$$W_G = O(4,2) \times G/O(4) \times O(2), \tag{1}$$

where G is the non-Abelian gauge group.

The formalism will be demonstrated for simplest quantity, - the vacuum-into-vacuum transition amplitude

$$\mathcal{Z}(\mathrm{u}) =< vac; \mathrm{u}|vac; \mathrm{u} >,$$

along the path $\mathrm{u}_\mu^a(x)$. Moreover, following to the idea that the calculation should be adjusted to the experiments ability [2], we will restrict ourselves calculating only the modulo squire

$$\mathcal{N}(\mathrm{u}) = | < vac; \mathrm{u}|vac; \mathrm{u} > |^2 = |\mathcal{Z}(\mathrm{u})|^2,$$

since, being the unmeasurable quantity, the phase of $\mathcal{Z}(\mathrm{u})$ is not important from physical point of view [4] (it is the principle of 'minimal necessity' in our terminology).

This quantity $\mathcal{N}(\mathrm{u})$ would normalize the observables and is equal to squire of the volume of $\mathcal{G}/\mathcal{H}$, see [5]. So, it defines a number of expected on the trajectory $\mathrm{u}_{\mu a}(x)$ degrees of freedom, i.e. $(\ln \mathcal{N}(\mathrm{u}))/2$ is proportional to the dimension of $\mathcal{G}/\mathcal{H}$. In the example (1):

$$\dim W_G = \dim G + 8 \tag{2}$$

since the $O(4) \times O(2)$-invariant solution $\mathrm{u}_{\mu a}(x)$ breaks both the gauge and the spatial symmetries. Last one includes the translational and spatial conformal transformations [3].

Having in consideration the probability-like quantity $\mathcal{N}(\mathrm{u})$, one can include into formalism the total probabilities conservation principle (see [2], where the role of unitarity condition in formation of quantum dynamics is described in details). So, one may prove that if we postulate the path-integral representation for $\mathcal{Z}(\mathrm{u})$, see (13) for scalar field case, and take into account the S-matrix unitarity condition then, if the canonical perturbation series exist (at least in Borel sense), $\mathcal{N}$ has the following strict path-integral representation:

$$\mathcal{N} = e^{-i\mathcal{K}(je)} \int DM_j(A) e^{-2iU(A,e)}. \tag{3}$$

In this expression $\mathcal{K}(je)$ acts as the differential operator of the auxiliary variables $j_{a\mu}$ and $e_{a\mu}$ at $j_{a\mu} = e_{a\mu} = 0$, see (15) and (84), and the expansion of $\exp\{-i\mathcal{K}\}$ generates the perturbation series. The functional $U(A, e)$ defines interaction. It may be expressed through the input classical action, see (16) and (85). The main point of our consideration is the differential measure DM_j since it is δ-like:

$$DM_j(A) = \prod_{a,\mu}\prod_x dA^{a\mu}(x)\delta\left(\frac{\delta S(A)}{\delta A^a_\mu(x)} + j^{a\mu}(x)\right), \tag{4}$$

where $S(A)$ is the *classical* Yang-Mills action. Notice that using the Fourier transform of functional δ-function in (4), one may easy find from (3) that $\mathcal{N}(\mathrm{u}) = |\mathcal{Z}(\mathrm{u})|^2$.

The structure of representation (3) did not depends on the dimension of system, concrete form of the Lagrangian and other 'local' properties of the theory. We will not repeat for this reason derivation of (3) since it is the same as in [2] (and [6], where the (1+1)-dimensional exactly integrable field theory was considered).

Following to the definitions of δ-function and operator $\mathcal{K}(je)$, one should start from the equation:

$$\frac{\delta S(A)}{\delta A^a_\mu(x)} = 0. \tag{5}$$

So, having a theory on the δ-like measure, we must consider [2] only the *strict* solution of Lagrange equation. Notice that the equation (5) has also the 'trivial' solution $A^a_\mu(x) = 0$, with the corresponding factor space W_0, $\dim W_0 = \dim G$, where G is the gauge group. The pQCD presents expansion around just this 'trivial' solution.

Then, if the general position concerning initial data is analyzed, we should neglect this 'trivial' solution since we will assume that our solution $\mathrm{u}^a_\mu(x)$ live in the factor space of $\dim(\mathcal{G}/\mathcal{H}) > \dim W_0$. This is a formal reason why the expansion in vicinity of $\mathrm{u}_{\mu a}(x) \neq const$ would be considered. Corresponding realization of the Yang-Mills theory would be the topological QCD (tQCD).

This selection rule [2] is our definition of the ground state. It should be stressed its importance. It says that first of all one should consider such solution of the Lagrange equation in the Minkowski space which is live in the factor space $\mathcal{G}/\mathcal{H}$ of highest dimension since, generally speaking, other orbits are realized on zero measure [7].

The extraordinary role of the factor space has specific explanation. At a first glance δ-likeness of measure (4) solves the problem of path integral calculation. But actually, to calculate the remaining integral in (3), measure (4) forces us to search new forms of perturbation theory. The formal reason is hidden in inhomogeneity of our Lagrange equation, see (4),

$$-\frac{\delta S(A)}{\delta A^a_\mu(x)} = j^{a\mu}(x). \tag{6}$$

So, the exact solutions of this equation are unknown even in the expansion over $j^{a\mu}(x)$ form if the corresponding homogeneous equation (5) has nontrivial solution $\mathrm{u}_{a\mu}(x) \neq$const.

Nevertheless one may try to solve this equation in the form of some perturbation series, expanding solution over $j^{a\mu}(x)$. This will lead to the theory which may have a near resemblance of the canonical one, see e.g. [9] where the 'straight pass' approximation was considered.

But the canonical perturbation theory for non-Abelian gauge theories have additional problems. First of all, the method of Faddeev-Popov [10], introduced for separation of dynamical degrees of freedom from pure gauge ones, in the most cases lead to the cumbersome perturbation theory with non-unitary ghost fields Lagrangian [11]. In the quantum gravity this, at first glance a technical complication, rise up to fundamental one, see e.g. [12].

Then, it was noted that it is impossible to fix the Coulomb gauge unambiguously for the Yang-Mills potentials of nontrivial topology [13]. Moreover, it was shown later that this conclusion did not depends on the chosen gauge, is general for non-Abelian gauge theories [14] if the expansion is builded around the nontrivial topology gauge orbits [15].

We will realize another approach to the problem. Namely, we will consider the mapping into the corresponding to $\mathrm{u}_{a\mu} = \mathrm{u}_{a\mu}(x; \xi, \eta, \lambda_a)$ factor space. Formally the mapping can be performed since the δ-like measure (4) defines the necessary and sufficient set of contributions into the functional integral. We will find the explicit form of $\mathcal{K}$, U and DM_j in the $\mathcal{G}/\mathcal{H}$ space. This is our first quantitative result.

Following to the idea formulated in [2], we will formulate the transformation in such a way that $\mathrm{u}_{a\mu} = \mathrm{u}_{a\mu}(x; \xi, \eta, \lambda_a)$ would be the generator of transformation:

$$\mathrm{u}_{a\mu} : \; A_{a\mu}(x) \rightarrow \; (\xi(t), \eta(t), \lambda_a(x)), \tag{7}$$

where the set $(\xi, \eta, \lambda_a) \in \mathcal{G}/\mathcal{H}$ will coincide at $j^{a\mu}(x) = 0$ with integration parameters of eq.(5), $\lambda_a(\mathbf{x})$ is the gauge phase and the variables ξ and η are the consequence of the spatial symmetry breaking. For the example (1), $\dim(\xi + \eta) = 8$. So, the combination of generators of violated by $\mathrm{u}_{a\mu}$ subgroup will be taken as the new quantum variables, instead of the Yang-Mills potentials $A_{\mu a}$. In other words, just the variables extracted by the Faddeev-Popov *anzats* as the 'non-physical' ones would be the dynamical variables of the tQCD.

The problem of definition and farther quantization of the factor space was solved in [2]. The method consist in formal mapping into the symplectic phase space W of the arbitrary high dimension, considering all dynamical variables of extended space as the q-numbers. It is the first step of calculations. Notice that the transformation always may be done canonically and the Jacobian of transformation would be equal to one. For this reason no ghost fields will appear.

Then the formalism allows to reduce W:

$$W = (\mathcal{G}/\mathcal{H}) \times R^* \tag{8}$$

This reduction of W up to $\mathcal{G}/\mathcal{H}$ is the second step of calculations. The realized transformation is singular since $\dim(\mathcal{G}/\mathcal{H}) < \dim W$. Nevertheless we will be able to extract corresponding artifact infinity equal to the volume of R^* and cancel it by the normalization.

The prove that the extracted by this way set of q-numbers is necessary and sufficient for quantization of the factor space $\mathcal{G}/\mathcal{H}$ will be crucial for our formalism. We will find that:

$$\mathcal{G}/\mathcal{H} = T^*V \times R, \tag{9}$$

where the quantum degrees of freedom only are belong to the cotangent symplectic manifold T^*V [16] and R is the c-number parameters subspace. The direct product (9) means that we will be able to isolate the quantum degrees of freedom from classical ones. So, it will be shown that $\lambda_a \in R$.

We will find that each order of the tQCD perturbation theory is transparently gauge invariant. This result seems natural since the gauge invariant quantity, the 'probability' $\mathcal{N}(\mathrm{u})$, is calculated. Therefore, there will not be a necessity to fix the gauge and, therefore, no 'copies' of Gribov [13] would arise. Moreover, it will be shown that no unphysical singularities connected to the Gribov's ambiguity [17] would occur in the formalism. This is our second quantitative result.

It is not hard to show, see also [2], that developed perturbation theory in the $\mathcal{G}/\mathcal{H}$ space presents expansion over $1/g$, where g is the interaction constant, and does not contain the terms $\sim g^n$, with $n > 0$. Such type of perturbation theory, over $1/g$, presents a definite problem from ordinary renormalization procedures point of view.

Indeed, the ordinary quantum field theory scheme assumes the multiplicative renormalization of the interaction constant: the renormalized constant $g_R = Z^{1/2}g < \infty$ and the renormalization factor $Z = \infty$ because of the ultraviolet divergences. Then, having the expansion over $1/g$, we come to evident contradiction: it is impossible to have the *infinite multiplicative* renormalizations in expansions over g and over $1/g$ simultaneously. For this reason this question would be considered in more details in our approach. We will show that our perturbation theory would not contain the divergences and the problem with renormalization would not arise. This is our third result.

It should be noted here that this results have been proposed to be obtained in [18] to distinguish the quantization on the factor space, but now this is done for complete perturbation theory. However to mention is that the quantitative progress was achieved taking into account the unitarity condition.

It was mentioned in [2] that our perturbation theory, over $1/g$, is dual to ordinary one, over g [19]. So, we may realize the expansion or over g, or over $1/g$, and the choice is defined only by the convenience. If the states counted by the expansion over g and over $1/g$ belong to orthogonal Hilbert spaces [20] then should not be any connections among terms of both expansion [2], only the result of summation of series should coincide. For this reason our formalism did not hides the contradiction: the expansion over g may contain divergences and it needs the renormalization, but the expansion over $1/g$ may be divergences free and no renormalizations would be necessary in it [21].

In the chosen way of calculations even the notion of *interacting* gluons in the Yang-Mills theory would disappeared (as well as the pQCD Feynman diagrams). Yet, we can not exclude the real (mass-shell) particles (gluons) emission [22] on the to-day level of understanding of abilities of our formalism and, therefore, we can not prove that the states counted in the expansion over g and over $1/g$ belong to the orthogonal Hilbert space. So, we will leave unsolved the problem of colored quanta emission since the question of confinement demands the more careful analysis.

The paper is organized as follows. Considering the solutions of Yang-Mills equation, one may use the *ansatz* [23]:

$$A^a_\mu(x) = \eta^a_{\mu\nu}\partial^\nu \ln\varphi(x), \tag{10}$$

where $\eta^a_{\mu\nu}$ are the real matrices. This ansatz reduce the Yang-Mills equation to the form [3]:

$$\partial^2\varphi + \kappa\varphi^3 = 0, \tag{11}$$

where κ is the integration constant. So, in Sec.2 we will formulate the ideology of mapping into the simpler factor space $W = O(4,2)/O(4)\times O(2)$ for scalar $O(4,2)$-invariant field theory with

the action:

$$S(\varphi)=\int d^4x\left(\frac{1}{2}(\partial_\mu\varphi)^2-\frac{\kappa}{4}\varphi^4\right). \tag{12}$$

In Sec.3 we will formulate the tQCD in the $\mathcal{G}/\mathcal{H}$ factor.

2 Scalar conformally invariant field theory

2.1 Definitions

We concentrate an attention in present section on calculation of $|\mathcal{Z}(\mathrm{u})|^2$, where

$$\mathcal{Z}=\int D\varphi e^{iS(\varphi)} \tag{13}$$

and $S(\varphi)$ is the action defined in (12)

As was explained, the integral

$$\mathcal{N}\equiv|\mathcal{Z}|^2=e^{-i\mathcal{K}(je)}\int DM_j(\varphi,\pi)e^{-2iU(\varphi,e)} \tag{14}$$

will be analyzed instead of (13). Here

$$2\mathcal{K}(je)=\mathrm{Re}\int_{C_+}dx\frac{\delta}{\delta j(x)}\frac{\delta}{\delta e(x)}\equiv\mathrm{Re}\int_{C_+}dx\hat{j}(x)\hat{e}(x). \tag{15}$$

At the very end of calculations one should take the auxiliary variables j and e equal to zero. The interactions are introduced by the functional

$$\begin{aligned}-2U(\varphi,e)&=S_{C_+}(\varphi+e)-S_{C_-}(\varphi-e)-2\mathrm{Re}\int_{C_+}d^4xe\frac{\delta S(\varphi)}{\delta\varphi}=\\&=2\kappa\mathrm{Re}\int_{C_+}dx\varphi(x)e^3(x)+O(\epsilon).\end{aligned} \tag{16}$$

The complex time formalism of Mills [24] was used and $S_{C_\pm}$ is the action defined on the complex time contour $C_\pm$. For sake of definiteness, we will use the complex time contours

$$C_\pm:\ t\to t\pm i\epsilon,\ \epsilon\to+0,\ |t|\leq\infty. \tag{17}$$

Let $\varphi_\pm$ are the fields on the $C_\pm$ branches of the Mills time contour and let $\partial C_\pm$ is the boundary of this branches. It was assumed the 'periodic' (closed-path [6]) boundary condition:

$$\varphi_+(t\in\partial C_+)=\varphi_-(t\in\partial C_-). \tag{18}$$

when the representation (14) was derived. This boundary condition should be maintained in the factor space.

Notice that considering the theory with Lagrangian (12), one may write $U(\varphi,e)$ in the following equivalent form (with $O(\epsilon)$ accuracy) :

$$3!U(\varphi,e)=-\int d^4xe(x)^3\frac{\delta^3}{\delta\varphi(x)^3}S(\varphi)=-\int d^4x\left\{e(x)\frac{\delta}{\delta\varphi(x)}\right\}^3S(\varphi), \tag{19}$$

This representation is useful for investigation of the perturbation theory symmetry properties. The indication that the contribution belongs to the Mills time contour was not shown in (19) since it was assumed that, for instance,

$$\frac{\delta j(t \in C_a)}{\delta j(t' \in C_b)} = \delta_{ab}\delta(t - t'),\ \ a, b = +, -. \tag{20}$$

For this reason it is sufficient to indicate the branch of Mills contour only in the definition of the operator (15).

We will consider the 'phase space' motion:

$$DM_j(\varphi, \pi) = \prod_x d\varphi(x) d\pi(x) \delta\left(\dot{\varphi}(x) - \frac{\delta H_j}{\delta \pi(x)}\right) \delta\left(\dot{\pi}(x) + \frac{\delta H_j}{\delta \varphi(x)}\right). \tag{21}$$

It is important that the formalism involves the *total* Hamiltonian

$$H_j = \int d^3x \left[\frac{1}{2}\pi^2 + \frac{1}{2}(\nabla\varphi)^2 + \frac{\kappa}{4}\varphi^4 - j\varphi\right] \tag{22}$$

and the last term $\sim j\varphi$ may be interpreted as the time dependent energy of random quantum excitations. It is evident that we may find the measure (9) if the first δ-function in (21) is used to calculate the integral over π. Thus, the representation (14), with the measure (21), may be considered as the 'first-order' formalism.

This ends the definition of the field theory on the Dirac measure.

2.2 Mapping into the factor space

Having a theory defined on the δ-like measure, arbitrary transformations are easily available. We will start from general situation introducing N fields $\{\xi(x), \eta(x)\}_N$, N is arbitrary.

To perform the transformation:

$$(\varphi(x), \pi(x)) \to \{\xi(x), \eta(x)\}_N \tag{23}$$

one should insert

$$1 = \frac{1}{\Delta(\varphi, \pi)} \int D\xi D\eta \prod_x \delta(F_\xi(\varphi, \pi; \xi, \eta)) \prod_x \delta(F_\eta(\varphi, \pi; \xi, \eta)) \tag{24}$$

into the integral (14). The functional δ-function $\prod_x \delta$ has following properties:

$$\int DX \prod_x \delta(X(x)) = 1,$$

$$\int DX \prod_x \delta(\partial_\mu X(x)) = \int \prod_x dX(x)\delta(\partial_\mu X(x)) = \int \prod_{x \neq x_\mu} dX_{(\mu)}(x). \tag{25}$$

Here $X_{(\mu)}(x)$ is the solution of equation $\partial_\mu X(x) = 0$, i.e. is the arbitrary, including constant, x_μ independent function.

Having the measure (21) and inserting the unit (24) into (14) the integrals of type:

$$\int D\xi D\eta D\varphi D\pi \Delta^{-1}(\varphi, \pi)\times$$

$$\times \prod \delta(F_\xi(\varphi, \pi; \xi, \eta))\delta(F_\eta(\varphi, \pi; \xi, \eta))\delta\left(\dot{\varphi} - \frac{\delta H_j}{\delta \pi}\right)\delta\left(\dot{\pi} + \frac{\delta H_j}{\delta \varphi}\right) \tag{26}$$

would appear. Notice that the $(\dim \xi + \dim \eta) = N$ was chosen arbitrary.

It is important that both measures in (26), over (ξ, η) and over (φ, π), are δ-like. This allows to change order of integration and integrate firstly over φ and π. It is natural, at first glance, to use for this purpose last two δ-functions. Then first ones will define the constraint. This scheme may restore the WKB perturbation theory, if the unite (24) is reduced to the Faddeev-Popov *ansatz* [2]. But if the first two δ-functions of (26) are used to calculate the integrals over φ and π, we perform transformation to the new dynamical variables (ξ, η). Then the last two δ-function will give the dynamical equations for (ξ, η). Both ways of computation would give the same result since one may use arbitrary δ-functions.

Thus, we wish to use the fact that the δ-like measure defines a complete set of contributions. Moreover, as follows from (14) and (15), the quantum perturbations, both in the $(\varphi, \pi) \in V$ and $(\xi, \eta) \in W$ spaces, would be generated by the same operator $\exp\{-i\mathcal{K}(je)\}$ and the interactions in both above cases are described by the same functional $U(u, e)$. This circumstances allows to describe the *quantum* dynamics in terms of new variables.

Then, if the 'phase space flow' (u, p) belongs to the manifold $\mathcal{G}/\mathcal{H}$ completely, we should be able to 'restore' it through the (u, p) flow. This is our key idea. We will see that this order of computation, inverse to ordinary one [25], mostly natural for us since it allows to start transformation from mostly general variables $(\xi, \eta) \in W$.

Following space-time local realization of the algebraic equations was offered in [2,6]:

$$F_\xi(\varphi, \pi; \xi, \eta) = \varphi(x) - u(x; \xi(x), \eta(x)) = 0,$$

$$F_\eta(\varphi, \pi; \xi, \eta) = \pi(x) - p(x; \xi(x), \eta(x)) = 0, \tag{27}$$

where $u = u(x; \xi(x), \eta(x))$, $p = p(x; \xi(x), \eta(x))$ are some *compound* functions. We will assume that this functions would be defined in accordance with our choice of $\mathcal{G}/\mathcal{H}$. The equalities (27) can be satisfied for arbitrary given $u(x; \xi(x), \eta(x))$, $p(x; \xi(x), \eta(x))$ and arbitrary N since integration over all $\varphi(x)$ and $\pi(x)$ is assumed.

Therefore, integral in (24) is not equal to zero since, generally speaking, always exist. The result of integration in (24) is denoted by $\Delta(\varphi, \pi)$ and in this sense the equality (24) is satisfied identically. The additional constraints for $u(x; \xi, \eta)$ and $p(x; \xi, \eta)$ will be offered later.

We will specify (27) adding the condition that the time dependence is hidden in $\xi(y, t)$ and $\eta(y, t)$, $x = (y, t)$, $\dim(y) = 3$. Thus, we would use, instead of (27), the equations:

$$\varphi(y, t) = u(y; \xi(y, t), \eta(y, t)),\ \pi(y, t) = p(y; \xi(y, t), \eta(y, t)). \tag{28}$$

In other aspects the functions $u(y; \xi, \eta)$, $p(y; \xi, \eta)$ for the time being are arbitrary. Notice that offered additional condition is evident since (u, p) would belong to $\mathcal{G}/\mathcal{H}$ completely. But, nevertheless, we will examine it [27]. Notice also the noncovariantness of equalities (28). This is a consequence of necessity to use the Hamiltonian formalism [2].

The integration measures in (26) over $\xi(y,t)$ and $\eta(y,t)$ are defined on the total Mills time contour $C = C_+ + C_-$:

$$\int_C dt = \int_{C_+ + C_-} dt = \int_{C_+} dt + \int_{C_-} dt, \tag{29}$$

and the integration should be performed with boundary condition (18):

$$u(;\xi(,t\in\partial C_+),\eta(,t\in\partial C_+)) = u(;\xi(,t\in\partial C_-),\eta(,t\in\partial C_-)). \tag{30}$$

Depending to the topology of the trajectory $u(;\xi,\eta)$, this boundary condition may lead to nontrivial consequences.

The mapping (28) is generated by the function u:

$$u:\ (\varphi,\pi)\to(\xi,\eta) \tag{31}$$

since the 'first-order' formalism is considered. It is important also to note that this transformation did not conserves the dimension:

$$\dim(\varphi,\pi)(y,t)\neq\dim(\xi,\eta)(y,t) \tag{32}$$

since $(\xi,\eta)\in\mathcal{G}/\mathcal{H}$ and $(\varphi,\pi)\in V$.

Proposition I. *Jacobian of transformation of the δ-like measure always can be done equal to one.*

Using first two δ-functions in (26) to perform integration over (φ,π) the Jacobian of the transformation (31) takes the form:

$$J = \frac{1}{\Delta(u,p)}\prod_{y,t}\delta\left(\dot{u}(y;\xi,\eta) - \frac{\delta H_j(u,p)}{\delta p(y;\xi,\eta)}\right)\delta\left(\dot{p}(y;\xi,\eta) + \frac{\delta H_j(u,p)}{\delta u(y;\xi,\eta)}\right), \tag{33}$$

where the definitions (27) and (28) was used. Notice that $\Delta = \Delta(u,p)$, as a result of integration over φ and π.

We should dioganalize arguments of remaining δ-functions. For this purpose following trick will be used [2]. So, for instance,

$$\delta\left(\dot{u} - \frac{\delta H_j}{\delta p}\right) = \delta\left(u_\xi\cdot\dot{\xi} + u_\eta\cdot\dot{\eta} - \frac{\delta H_j}{\delta p}\right) =$$

$$= \delta\left(u_\xi\cdot\left\{\dot{\xi} - \frac{\delta h_j}{\delta\eta}\right\} + u_\eta\cdot\left\{\dot{\eta} + \frac{\delta h_j}{\delta\xi}\right\} + u_\xi\cdot\frac{\delta h_j}{\delta\eta} - u_\eta\cdot\frac{\delta h_j}{\delta\xi} - \frac{\delta H_j}{\delta p}\right),$$

where $u_X\equiv\partial u/\partial X$, $X=\xi,\eta$ and $h_j = h_j(\xi,\eta)$ is the auxiliary functional. Let us choose it by the equality:

$$u_\xi\cdot\frac{\delta h_j}{\delta\eta} - u_\eta\cdot\frac{\delta h_j}{\delta\xi} - \frac{\delta H_j}{\delta p} = \frac{\partial u}{\partial\xi}\cdot\frac{\delta h_j}{\delta\eta} - \frac{\partial u}{\partial\eta}\cdot\frac{\delta h_j}{\delta\xi} - \frac{\delta H_j}{\delta p} = \{u,h_j\} - \frac{\delta H_j}{\delta p} = 0, \tag{34}$$

where $\{,\}$ is the Poisson bracket. The scalar product means that the sets $\{\xi\}$ and $\{\eta\}$ was ordered in such a way that the Poisson bracket would be well defined. This ordering always possible iff W is the symplectic manifold.

Then, if (34) is satisfied,

$$\delta\left(\dot{u} - \frac{\delta H_j}{\delta p}\right) = \delta\left(u_\xi \left\{\dot{\xi} - \frac{\delta h_j}{\delta \eta}\right\} + u_\eta \left\{\dot{\eta} + \frac{\delta h_j}{\delta \xi}\right\}\right),$$

The analogous expression one may find for second δ-function:

$$\delta\left(\dot{p} + \frac{\delta H_j}{\delta u}\right) = \delta\left(p_\xi \left\{\dot{\xi} - \frac{\delta h_j}{\delta \eta}\right\} + p_\eta \left\{\dot{\eta} + \frac{\delta h_j}{\delta \xi}\right\}\right),$$

and h_j and p should obey additional to (34) equality:

$$\{p, h_j\} + \frac{\delta H_j}{\delta u} = 0. \tag{35}$$

On this stage two equality (34) and (35) are the equations for functions $u(;\xi,\eta)$, $p(;\xi,\eta)$ and $h_j(\xi,\eta)$. Thus, being vague, this mechanism of mapping is able to endure more constraints.

Using the ordinary property of δ-function:

$$\delta(a - b) = \int dc\delta(c - a)\delta(c - b),$$

we can write that:

$$J(\xi,\eta) = \frac{1}{\Delta(u,p)} \int D\xi' D\eta' \prod_x \delta(u_\xi \cdot \xi' + u_\eta \cdot \eta')\delta(p_\xi \cdot \xi' + p_\eta \cdot \eta') \times$$
$$\times\, \delta\left(\xi' - \left\{\dot{\xi} - \frac{\delta h_j}{\delta \eta}\right\}\right) \delta\left(\eta' - \left\{\dot{\eta} + \frac{\delta h_j}{\delta \xi}\right\}\right). \tag{36}$$

Let us assume that the functional integral $\Delta(u,p)$ may be written in the form:

$$\Delta(u,p) =$$
$$= \int D\xi' D\eta' \prod_{y,t} \delta(\varphi(y,t) - u(y;\xi+\xi',\eta+\eta'))\delta(\pi(y,t) - p(y;\xi+\xi',\eta+\eta')) =$$
$$= \int D\xi' D\eta' \prod_{y,t} \delta(u_\xi \xi' + u_\eta \eta')\delta(p_\xi \xi' + p_\eta \eta') \neq 0 \tag{37}$$

This is possible since the functions $\varphi(y,t)$ and $\pi(y,t)$ was chosen in such a way that the equalities (27) are satisfied. The inequality (37) excludes the degeneracy. For this reason only $\xi' = \eta' = 0$ are essential in the integral (37).

In result the determinant $\Delta(u,p)$ is canceled identically:

$$DM_j(\xi,\eta) = \prod_{y,t} d\xi(y,t)\eta(y,t)\delta\left(\dot{\xi}(y,t) - \frac{\delta h_j}{\delta \eta(y,t)}\right) \delta\left(\dot{\eta}(y,t) + \frac{\delta h_j}{\delta \xi(y,t)}\right) \tag{38}$$

since one may leave arbitrary pare of δ-functions in (36) and $\xi' = \eta' = 0$ are essential. Therefore, because of cancelation of the functional determinants our perturbation theory would be free

from the ghost fields. This considerably simplifies the described formalism. Notice that the equalities (34), (35) and (37) should be satisfied to have this result.

The transformed measure (38) depends on the auxiliary functional $h_j = h_j(\xi, \eta)$, defined by the equalities (34) and (35). So, choosing *arbitrary* $u(;\xi, \eta)$ and $p(;\xi, \eta)$ with the property (37), one may find h_j from (34) and (35), and then (38) would be the transformed measure.

Therefore, mapping (31) based on the equations (34) and (35) admits one more equation for $u(;\xi, \eta)$, $p(;\xi, \eta)$ and $h_j(\xi, \eta)$. We will consider following example in present paper. One may note from (38) that h_j has a meaning of transformed Hamiltonian of new equations:

$$\dot{\xi}(y,t) = \frac{\delta h_j(\xi,\eta)}{\delta \eta(y,t)}, \ \dot{\eta}(y,t) = -\frac{\delta h_j(\xi,\eta)}{\delta \xi(y,t)}. \tag{39}$$

Proposition II. *If*

$$h_j(\xi,\eta) = H_j(u,p), \tag{40}$$

then the Poisson equations (34), (35) would define the 'phase space flow' (u,p).

Indeed, having in mind (28),

$$\dot{u} = u_\xi \dot{\xi} + u_\eta \dot{\eta} = u_\xi \frac{\delta h_j}{\delta \eta} - u_\eta \frac{\delta h_j}{\delta \xi} = \{u, h_j\} = \frac{\delta H_j}{\delta p}, \tag{41}$$

where (39) and then (34) were used. The same equation one may find for p:

$$\dot{p} = p_\xi \dot{\xi} + p_\eta \dot{\eta} = p_\xi \frac{\delta h_j}{\delta \eta} - p_\eta \frac{\delta h_j}{\delta \xi} = \{p, h_j\} = -\frac{\delta H_j}{\delta u}. \tag{42}$$

Therefore, having (40) the equations (34), (35), simultaneously with (39), are equal to the Hamiltonian equations (41) and (42). Notice also that in this case the time dependence actually should be hidden into ξ and η.

It should be stressed also that as follows (41) and (42) fixed by (34), (35) and completed by (40) and (37) transformations unique in those respects that other 'type' of mapping would lead to 'unnatural', much more complicate, formalism.

Having (34), (35), (40) and taking into account (37), we get to the 'overdetermined' system of constraints, which may be inconsistent. The Coulomb problem gives quantum mechanical example of such system [2]. At all evidence, the $O(4) \times O(2)$-invariant solution did not obey (37) also. On other hand, if we reject (37) then the determinant $\Delta(u, p)$ is not canceled and the formalism would contain the ghosts.

2.3 Structure of dual perturbation theory

The problem of mapping for the degenerate case was solved in [2]. It was assumed that one may 'softly' take off the degeneracy, i.e. exist some parameter $\varepsilon \to 0$ which regulates the strength of degeneracy breaking and at $\varepsilon = 0$ we have the degenerate limit [26]. Following proposition will be important in this connection.

Proposition III. *The quantum perturbation conserves the topology of phase space flow.*

Indeed, notice that the equations (34) and (35) should be satisfied for arbitrary $j(y, t)$. Let us consider the consequence of this proposition. Remembering (22), and using the definition

(40), we find that (34) at $j=0$ gives equality:

$$\{u_\xi p_\eta - u_\eta p_\xi - 1\}\,\frac{\delta H}{\delta p(y,t)} = \{u_\eta u_\xi - u_\xi u_\eta\}\,\frac{\partial H}{\partial u(y,t)},\quad H = H_j|_{j=0}\,.$$

Here u and p are the compound functions of $\xi=\xi(y,t)$ and $\eta=\eta(y,t)$. This equality is identically satisfied if the space-time local Poison brackets:

$$\{u(y,t),p(y,t)\}=1,\quad \{u(y,t),u(y,t)\}=0 \tag{43}$$

are satisfied. The equation (35) at $j=0$ adds following conditions:

$$\{u(y,t),p(y,t)\}=1,\quad \{p(y,t),p(y,t)\}=0 \tag{44}$$

It is not hard to see that the higher orders over j did not give new conditions, i.e. the Poison algebra, completed by (40), is closed. In other words, the quantum perturbations conserve the topology [28] of the phase space flow.

The proposition **III** means that the quantum perturbations would not alter the structure of $u=u(;\xi,\eta)$ and $p=p(;\xi,\eta)$ and they are solution of *classical* (homogeneous) equations:

$$\{u(y;\xi,\eta),h(\xi,\eta)\} = \frac{\delta H(u,p)}{\delta p(y;\xi,\eta)},\ \{p(y;\xi,\eta),h(\xi,\eta)\} = -\frac{\delta H(u,p)}{\delta u(y;\xi,\eta)}. \tag{45}$$

The j dependence is defined by the equations (39) and is confined completely in ξ and η only.

So, we may start from a theory with generalized Hamiltonian:

$$h_j(\xi,\eta) = H_j(u,p) + \varepsilon\tilde{H}_j(u,p), \tag{46}$$

where the additive term $\sim\varepsilon\to 0$. This proposition means that the 'direct' mechanism of degeneracy breaking is considered [26] and the Hamiltonian $h_j(\xi,\eta)$ may be chosen in such a way that some of *derivatives* over auxiliary (artificial) fields ξ' and η' have a property:

$$u_{\xi'}\sim u_{\eta'}\sim p_{\xi'}\sim p_{\eta'}\sim\varepsilon\to 0,\quad (\xi',\eta')\in R^*. \tag{47}$$

This is enough to formulate conserving the phase space volume transformation of quantum theory.

Thus, we start from the variables $(\xi,\eta)\in W$ and scalar functions $u=u(y;\xi,\eta)$, $p=p(y;\xi,\eta)$. They should obey the inequality (37) and define the functional $h_j(\xi,\eta)$ through the equations (34), (35). This allows to cancel the determinant $\Delta(u,p)$. Then we extract the auxiliary variables ξ' and η' assuming (47). This will allow to exclude the auxiliary variables and should reduce the system to physical one. The physical content of this procedure was described in [2].

Following property of the perturbation theory in the W space will be used to realize this program of reduction. In result of our mapping the integral $\mathcal{N}$ takes the form:

$$\mathcal{N}(\mathrm{u}) = e^{-i\mathcal{K}(je)}\int DM_j(\xi,\eta)e^{-2iU(u,e)}, \tag{48}$$

where $DM_j(\xi,\eta)$ is defined in (38). Notice that in this expression U depends on $u=u(y;\xi,\eta)$.

It was shown in [2] that the mapped representation (48) allows to split the 'quantum force' $j(y,t)$ and corresponding 'virtual field' $e(y,t)$ on the projection on the axes of W. It is easy to find the result of this procedure:

$$2\mathcal{K}(je) = \mathrm{Re} \int_{C_+} d^3x dt \left\{ \hat{j}_\xi(y,t) \cdot \hat{e}_\xi(y,t) + \hat{j}_\eta(y,t) \cdot e_\eta(y,t) \right\} \tag{49}$$

and

$$e = e_\xi \cdot \frac{\partial u}{\partial \eta} - e_\eta \cdot \frac{\partial u}{\partial \xi}. \tag{50}$$

The hat symbol in (49) means the derivative over corresponding quantity. At the very end of calculation one should take $j_X = e_X = 0$, $X = (\xi, \eta)$. The scalar product means summation over all components of ξ and η.

Inserting (50) into (19) one can find that

$$-3!U(u,e) = \int d^3x dt \left\{ e_\xi \cdot \frac{\partial u}{\partial \eta} \frac{\delta}{\delta u} - e_\eta \cdot \frac{\partial u}{\partial \xi} \frac{\delta}{\delta u} \right\}^3 S(u) =$$

$$= \int d^3x dt \left\{ e_\xi \cdot \frac{\partial u}{\partial \eta} \frac{\partial}{\partial u} - e_\eta \cdot \frac{\partial u}{\partial \xi} \frac{\partial}{\partial u} \right\}^3 \mathcal{L}(u), \tag{51}$$

where $\mathcal{L}(u)$ is the Lagrangian density. This shows that the interaction functional $U(u,e)$ has the symmetry properties of the Lagrangian density.

Formally new perturbation generating operator (49) gives the same perturbation series, but with the rearranged sequence of terms, i.e. the splitting of j did not change the 'convergence' of the perturbation series (over $1/\kappa$ since $u \sim 1/\sqrt{k}$). At the same time, this splitting of the source j is useful since allows to analyze the excitation of each degree of freedom, i.e. of components of the phase space flow along the axis of W, independently.

Noting that e_X, $X = \xi, \eta$, is conjugate to j_X, it is easy to conclude that the action of the operator (49) leads to the operator

$$\left\{ \frac{\delta}{\delta j_\xi} \cdot \frac{\partial u}{\partial \eta} \frac{\partial}{\partial u} - \frac{\delta}{\delta j_\eta} \cdot \frac{\partial u}{\partial \xi} \frac{\partial}{\partial u} \right\} \sim \{\hat{j} \wedge \hat{X}\}.$$

This operator is the invariant of canonical transformations. If by some reason $d\omega_X^2 = \hat{j}_X \wedge \hat{X} = 0$, then the motion along the X-th axis will be classical. This is the mechanism of reduction of the quantum degrees of freedom. Firstly this important properties of our formalism was described in [2]. We will continue this question in Sec.2.4.

Proposition IV. *New fields ξ and η can not depend on the coordinate y if the scalar theory is considered*, i.e.

$$\xi = \xi(t), \quad \eta = \eta(t), \tag{52}$$

for scalar theory (12).

This conclusion follows from proposition **III**. The reason is that the dynamical problem was divided on two parts. First part of the problem consist in solution of the *classical* equations (45). It defines a structure of the compound functions $u(y;\xi,\eta)$ and $p(y;\xi,\eta)$. The second part consist in definition of the *time* dependence of (ξ,η) through the equations (39) and

(40). Finally, if (ξ, η) in zero order over $j(y,t)$ are the y independent parameters, the quantum perturbations are unable to change this property.

It is noticeable that if $\xi = \xi(t)$ and $\eta = \eta(t)$ then we will find from (34) and (35), instead of (43) and (44), the *canonical* equal-time commutator relations:

$$\{u(y;\xi(t),\eta(t)), p(y';\xi(t),\eta(t))\} = \delta(y-y'). \tag{53}$$

Thus, our quantization scheme would restore the canonical one in the factor space W. In this sense the independence of ξ and η from y is natural.

Nevertheless it seems useful to demonstrate the proposition **IV** explicitly. The elements (49) and (38) are used in Appendix A to demonstrate the reduction:

$$(\xi,\eta)(y,t) \to (\xi,\eta)(t). \tag{54}$$

This involves reduction of the operators:

$$(\hat{j}_X, \hat{e}_X)(y,t) \to (\hat{j}_X, \hat{e}_X)(t),\ \ X = \xi, \eta. \tag{55}$$

The structure of corresponding perturbation theory is described in subsequent subsection.

2.4 Reduction

Therefore, for considered scalar theory,

$$2\mathcal{K}(je) = \mathrm{Re}\int_{C_+} dt\left\{\hat{j}_\xi(t)\cdot\hat{e}_\xi(t) + \hat{j}_\eta(t)\cdot e_\eta(t)\right\} \tag{56}$$

and

$$e(y;\xi(t),\eta(t)) = e_\xi(t)\cdot\frac{\partial u(y;\xi(t),\eta(t))}{\partial\eta(t)} - e_\eta(t)\cdot\frac{\partial u(y;\xi(t),\eta(t))}{\partial\xi(t)}. \tag{57}$$

The result of disappearance of the y dependencies in ξ and η is reduction of the field-theoretical problem to the quantum mechanical one. So, $L(u) = V(\xi,\eta)$ play here the role of the mechanical potential for a particle with the *phase space* coordinate (ξ,η).

The measure takes the form:

$$DM_j(\xi,\eta) = \prod_t d\xi(t)d\eta(t)\delta(\dot{\xi}(t) - \omega_\eta(\xi,\eta) - j_\xi(t))\delta(\dot{\eta}(t) + \omega_\xi(\xi,\eta) - j_\eta(t)), \tag{58}$$

where the 'velocity'

$$\omega_X(\xi,\eta) = \frac{\partial h(\xi,\eta)}{\partial X}. \tag{59}$$

Let us remember now the definition (47):

$$u = u(y;\xi(t),\eta(t);\varepsilon\xi'(t),\varepsilon\eta'(t)),\quad \varepsilon\to 0, \tag{60}$$

where

$$\dim\xi = n,\ \ \dim\eta = m,\ \ \dim(\xi+\xi') = \dim(\eta+\eta') = N. \tag{61}$$

Inserting (60) into Lagrangian, we find that:

$$L(u) = \int d^3x \mathcal{L}(u(y; \xi(t), \eta(t))) + O(\varepsilon). \tag{62}$$

We are able now to define the dimension of T^*V taking

$$N = \dim(\mathcal{G}/\mathcal{H}). \tag{63}$$

So, $N = 8$ for the example (1).

Proposition V. *If we have (60) and (61) then*

$$\dim T^*V = \min\{n, m\} \tag{64}$$

Let us consider following three possibilities to demonstrate this proposition.

(**a**). $n = m$, $N = 2n$.

In this case the interaction functional $U(u, e)$ takes the form:

$$\begin{aligned} -3!U(u,e) &= \int dt \left\{ \left(e_x \cdot \frac{\partial}{\partial \eta} - e_\eta \cdot \frac{\partial}{\partial \xi} \right)_n + \right. \\ &+ \left. \left(e_{x'} \cdot \frac{\partial}{\partial \eta'} - e_{\eta'} \cdot \frac{\partial}{\partial \xi'} \right)_{N-n} \right\}^3 L(u) = \\ &= \int dt \left\{ \left(e_x \cdot \frac{\partial}{\partial \eta} - e_\eta \cdot \frac{\partial}{\partial \xi} \right)_n \right\}^3 L(u), \end{aligned} \tag{65}$$

where (62) was used. The index n means that the scalar products include n terms, and N may be chosen equal to n. The measure

$$DM_j(\xi, \eta) = \prod_t d^n\xi(t) d^n\eta(t) \delta^{(n)}(\dot{\xi} - \omega_\eta - j_\xi) \delta^{(n)}(\dot{\eta} + \omega_\xi - j_\eta).$$

(**b**). $n > m$, $N = n + m$.

In this case

$$-3!U(u,e) = \int dt \left\{ \left(e_x \cdot \frac{\partial}{\partial \eta} - e_\eta \cdot \frac{\partial}{\partial \xi} \right)_m + \left(e'_\eta \cdot \frac{\partial}{\partial \xi} \right)_{(n-m)} \right\}^3 V(\xi, \eta), \tag{66}$$

since η' is absent in $V(\xi, \eta)$. Therefore, e'_η has only the $(n - m)$ components.

The measure takes the form:

$$DM_j(\xi, \eta) = \prod_t d^n\xi(t) d^m\eta(t) d^{(n-m)}\eta'(t)$$

$$\delta^{(m)}(\dot{\xi} - \omega_\eta - j_\xi) \delta^{(m)}(\dot{\eta} + \omega_\xi - j_\eta) \delta^{(n-m)}(\dot{\xi} - j_\xi) \delta^{(n-m)}(\dot{\eta}' + \omega_\xi - j_{\eta'})$$

since $N = (n + m)$. Notice that η' is contained only in the argument of last δ-function. For this reason we always can perform the shift: $\dot{\eta}' \to \dot{\eta}' - \omega_\xi + j_{\eta'}$. In result:

$$DM_j(\xi, \eta) =$$

$$= \prod_t d^n\xi(t)\eta^m(t)\delta^{(m)}(\dot{\xi} - \omega_\eta - j_\xi)\delta^{(m)}(\dot{\eta} + \omega_\xi - j_\eta)\delta^{(n-m)}(\dot{\xi} - j_\xi)\delta^{(n-m)}(\dot{\eta}')$$

and the $j_{\eta'}$ dependence is disappeared. For this reason the $\hat{j}_{\eta'}$ dependence in the operator $\mathcal{K}$ may be omitted. In result,

$$2\mathcal{K}(je) = \mathrm{Re}\int_{C_+} dt \left\{(\hat{j}_\xi \cdot \hat{e}_\xi)_m + (\hat{j}_\eta \cdot \hat{e}_\eta)_m + (\hat{j}_\xi \cdot \hat{e}_\xi)_{(n-m)}\right\}.$$

There is not operator $\hat{e}'_\eta$ and, for this reason, one should take $e_{\eta'}$ equal to zero. Therefore,

$$-3!U(u,e) = \int dt \left\{e_x \cdot \frac{\partial}{\partial\eta} - e_\eta \cdot \frac{\partial}{\partial\xi}\right\}_m^3 V(\xi,\eta) \tag{67}$$

and the $(n-m)$ components of e_ξ and j_ξ may be taken equal to zero everywhere:

$$2\mathcal{K}(je) = \mathrm{Re}\int_{C_+} dt \left\{\hat{j}_\xi \cdot \hat{e}_\xi + \hat{j}_\eta \cdot \hat{e}_\eta\right\}_m. \tag{68}$$

Accordingly,

$$DM_j(\xi,\eta) = dR\prod_t d^m\xi(t)d^m\eta(t)\delta^{(m)}(\dot{\xi} - \omega_\eta - j_\xi)\delta^{(m)}(\dot{\eta} + \omega_\xi - j_\eta), \tag{69}$$

where

$$dR = d^{(N-2m)}\xi(0) \tag{70}$$

is the element of R. The trivial auxiliary elements was omitted.

The same analyses may be done for the case $n < m$.

In result, assuming that η is the 'action' variable,

$$\omega_\eta = \omega(\eta) \equiv \partial h(\eta)/\partial\eta,\ \omega_\xi = 0,$$

we can write:

$$DM_j(\xi,\eta) = dR\prod_{i=1}^{\min\{m,n\}}\prod_t d\xi_i(t)\eta_i(t)\delta(\dot{\xi}_i - \omega_i(\eta) - j_{i\xi})\delta(\dot{\eta}_i - j_{i\eta}). \tag{71}$$

Therefore,

$$W = T^*V \times R \tag{72}$$

and dR is the differential measure of the subspace R.

This ends the prove of proposition **V**.

So, the equations for ξ and η take the form:

$$\dot{\xi}(t) = \omega(\eta) + j_\xi(t),\ \dot{\eta}(t) = j_\eta(t) \tag{73}$$

The second equation is simply integrable:

$$\eta(t) = \eta_0 + \int dt' g(t-t')j_\eta(t') \equiv \eta_0 + \eta_j(t). \tag{74}$$

Inserting this solution into the first equation in (73) one may find:

$$\xi(t) = \xi_0 + \int dt' g(t-t')\omega(\eta_0 + \eta_j(t')) + \int dt' g(t-t') j_\xi(t') \equiv \xi_0 + \bar{\omega}_j(t)t + \xi_j(t), \tag{75}$$

where the abbreviation:

$$\bar{\omega}(t)t = \int dt' g(t-t')\omega(\eta_0 + \eta_j(t')) \tag{76}$$

was used. The Green function $g(t-t')$ was defined in [2]:

$$g(t-t') = \Theta(t-t'), \tag{77}$$

where $\Theta(t-t')$ is the step function with boundary property:

$$\Theta(0) = 1. \tag{78}$$

In result,

$$u = u(y; \xi_0 + \bar{\omega}_j(t)t + \xi_j, \eta_0 + \eta_j) \tag{79}$$

and the term

$$\sim \frac{1}{n!}\{-2i\mathcal{U}(u,j)\}^n = O(\frac{1}{\kappa^n})$$

gives the n-th order of our perturbation theory over $1/\kappa$ since $u = O(1/\sqrt{\kappa})$.

3 Non-Abelian gauge field theory

3.1 Yang-Mills theory on Dirac measure

The action of considered theory

$$S(A) = \frac{1}{2g}\int d^4x F_{\mu\nu a}(A) F^{\mu\nu}_a(A) \tag{80}$$

is the $O(4,2)$ invariant and the Yang-Mills fields

$$F_{\mu\nu a}(A) = \partial_\mu A_{\nu a} - \partial_\nu A_{\mu a} - C^{bc}_a A_{\mu b} A_{\nu c} \tag{81}$$

are the covariant of non-Abelian gauge transformations. The gauge group will not be specified.

We will consider the integral

$$\mathcal{N} = e^{-i\mathcal{K}(je)} \int DM_j e^{-2iU(A,e)}, \tag{82}$$

where the measure

$$DM_j(A) = \prod_{\mu,a}\prod_x dA^a_\mu(x,t)\delta(D^{\nu b}_a F_{\nu\mu b} - j_{\mu a}) \tag{83}$$

is manifestly conformal and gauge invariant if $j_{\mu a} = 0$. The covariant derivative

$$D^{\mu b}_a = \partial^\mu \delta^b_a + C^{bc}_a A^\mu_c.$$

The perturbations generating operator

$$2\mathcal{K}(je) = \mathrm{Re}\int_{C_+} d^4x \frac{\delta}{\delta j_a^\mu(x,t)}\frac{\delta}{\delta e_{\mu a}(x,t)} \equiv \mathrm{Re}\int_{C_+} d^4x \hat{j}_{\mu a}(x,t)\hat{e}_a^\mu(x,t), \tag{84}$$

The auxiliary variables $j_{\mu a}$ and e_a^μ should be taken equal to zero at the very end of calculations. The functional

$$-2U(A,e) = (S_{C_+}(A+e) - S_{C_-}(A-e)) - 2\mathrm{Re}\int_{C_+} d^4x e_a^\mu(x)\frac{\delta S(A)}{\delta A_a^\mu} + O(\varepsilon) \tag{85}$$

describes interactions. All above quantities are defined on the Mills time contours

$$C_\pm: \ t \to t \pm i\epsilon, \ \epsilon \to +0, \ |t| \leq \infty. \tag{86}$$

This gives the rule as to avoid the light-cone singularities solving the equation:

$$D_a^{\nu b} F_{\nu\mu b} = j_{\mu a}. \tag{87}$$

One can omit in (85) terms $\sim \epsilon \to +0$. Therefore, $U(A,e) = O(e^3)$ and may contain only the odd powers of $e_{a\mu}$. This means that we may write $U(A,e)$ in the form:

$$U(A,e) = -\int d^4x \left\{ e_a^\mu(x) \frac{\delta}{\delta A_a^\mu(x)} \right\}^3 S(A), \tag{88}$$

see (19).

3.2 First-order formalism

The noncovariant first order formulation in terms of the 'electric' field

$$E_a^i = F_a^{i0}, \tag{89}$$

presents introduction into the necessary for us Hamiltonian description. The action in this term has the form

$$S_{C_\pm}(A,F) = \frac{1}{g}\int_{C_\pm} d^4x \left\{ \dot{\mathbf{A}}_a \cdot \mathbf{E}_a + \frac{1}{2}(\mathbf{E}_a^2 + \mathbf{B}_a^2(\mathbf{A})) - A_{0a}(\mathbf{D}\cdot\mathbf{E})_a \right\}, \tag{90}$$

where the 'magnetic' field

$$B_{ia}(\mathbf{A}) = (rot\mathbf{A})_{ia} + \frac{1}{2}\varepsilon_{ijk}[A_j, A_k]_a \tag{91}$$

is not the independent quantity and was introduce to shorten the formulae. Notice that A_{0a} did not contain the conjugate pare and the action S is linear over it.

The measure (83) may be written in the first-order formalism representation ($d\mathbf{A}_a = \prod_i dA_{ia}$):

$$DM_j(\mathbf{A},\mathbf{P}) = \prod_{a,i}\prod_x d\mathbf{A}_{ai}(x) d\mathbf{P}_{ai}(x)\delta(\mathbf{D}_a^b \cdot \mathbf{P}_b)\times$$

$$\times\,\delta\left(\dot{\mathbf{P}}_a(x)+\frac{\delta H_j(\mathbf{A},\mathbf{P})}{\delta \mathbf{A}_a(x)}\right)\delta\left(\dot{\mathbf{A}}_a(x)-\frac{\delta H_j(\mathbf{A},\mathbf{P})}{\delta \mathbf{P}_a(x)}\right), \tag{92}$$

where $H_j(\mathbf{A},\mathbf{P})$ is the total Hamiltonian:

$$H_j=\frac{1}{2g}\int d^3x\left(\mathbf{P}_a^2+\mathbf{B}_a^2(\mathbf{A})\right)+\int d^3x\mathbf{j}_a\mathbf{A}_a, \tag{93}$$

$\mathbf{P}_a(x)\equiv\mathbf{E}_a(x)$ is the conjugate to $\mathbf{A}_a(x)$ momentum and $\mathbf{B}_a(\mathbf{A})$ was defined in (91). We may introduce into DM_j additional δ-function:

$$\prod_a\prod_x\delta(\mathbf{B}_a^i-(\mathrm{rot}\mathbf{A})_a^i-\frac{1}{2}\varepsilon_{jk}^i[A^j,A^k]_a). \tag{94}$$

Then the Hamiltonian in (93) becomes symmetric over electric $\mathbf{E}_a$ and magnetic $\mathbf{B}_a$ fields.

Notice that the first δ-function in (92) is the consequence of linearity of the action over A_{0a}. The time component A_{0a} has the meaning of Lagrange multiplier for the Gauss law:

$$\mathbf{D}_a^b\cdot\mathbf{P}_b=0. \tag{95}$$

It should be stressed that there is not equation for the time component A_{0a}. Moreover, the A_{0a} dependence was completely disappeared from formalism since the interaction functional $U(A,e)$ is defined by the third derivative over $A_{\mu a}$, see (88).

3.3 Mapping into the factor space

The measure (92) is not physical since it contains three (for given a) vector potential $\mathbf{A}_a(x)$. To exclude the unphysical degree of freedom, the gauge fixing Faddeev-Popov *ansatz* is oftenly used. But we will consider, as was described in previous section, another approach.

We will introduce the functional

$$\Delta(A,P)=$$
$$=\int D\xi D\eta\prod_a\delta\left(\mathbf{A}_a(x)-\mathbf{u}_a(x;\xi(x),\eta(x))\right)\delta\left(\mathbf{P}_a(x)-\mathbf{p}_a(x;\xi(x),\eta(x))\right) \tag{96}$$

to realize the transformation

$$u:\ (A,P)_a(x)\rightarrow(\xi,\eta)(x), \tag{97}$$

to the compound vector functions $(\mathbf{u},\mathbf{p})_a(x;\xi(x),\eta(x))$ of the space-time local parameters $(\xi,\eta)(x)$. It is assumed that $\Delta\neq 0$.

Performing transformation (97), we find:

$$DM_j(\xi,\eta)=\frac{1}{\Delta_c(u)}\prod_a\prod_x d\xi d\eta d\lambda_a dq_a\delta(\mathbf{D}_a^b\cdot\mathbf{p}_b)\times$$
$$\times\,\delta\left(\dot{\mathbf{u}}_a(x)-\frac{\delta H_j}{\delta\mathbf{p}_a(x)}\right)\delta\left(\dot{\mathbf{p}}_a(x)+\frac{\delta H_j}{\delta\mathbf{u}_a(x)}\right). \tag{98}$$

Here the gauge phase λ_a and conjugate to it q_a was extracted from the set of variables ξ and η.

Using the result of previous section, one may dioganalize arguments of δ-functions. In result:

$$DM_j(\xi,\eta,\lambda,Q)=\prod_{x,t,a} d\xi d\eta d\lambda dq\delta\left(\mathbf{D}_a^b(\mathbf{u})\cdot\mathbf{p}_b\right)\delta\left(\dot{\lambda}_a-\frac{\delta h_j}{\delta q_a}\right)\delta\left(\dot{q}_a+\frac{\delta h_j}{\delta\lambda_a}\right)\times$$

$$\times\,\delta\left(\dot{\xi}-\frac{\partial h_j}{\partial\eta}\right)\delta\left(\dot{\eta}+\frac{\partial h_j}{\partial\xi}\right). \tag{99}$$

The equality (99) is hold iff h_j is defined by Poisson equations (for the 3-vectors given $\mathbf{u}_a$ and $\mathbf{p}_a$):

$$\{\mathbf{u}_a(x),h_j\}=\frac{\delta H_j}{\delta\mathbf{p}_a(x)},\ \ \{\mathbf{p}_a(x),h_j\}=-\frac{\delta H_j}{\delta\mathbf{u}_a(x)} \tag{100}$$

considering (ξ,η) and (λ,q) in the Poisson brackets as the canonically conjugate pares.

If we add to (100) one more equation:

$$h_j(\xi,\eta,\lambda,q)=H_j(\mathbf{u}_a,\mathbf{p}_a) \tag{101}$$

then, as was shown in previous section, $\mathbf{u}_a$ and $\mathbf{p}_a$ should be solution of incident equations, assuming that (100) are hold on the measure (99). Then

$$\mathbf{D}_a^b(u)\cdot\mathbf{p}_b\equiv 0 \tag{102}$$

since $\mathbf{p}_b$ is the solution of eq.(100) at arbitrary $j_{\mu a}$. This remarkable result is the consequence of mapping into the invariant space $\mathcal{G}/\mathcal{H}$ to which the classical flow belongs completely. Therefore, corresponding δ-function in (111) gives identically

$$\prod_x\delta(0).$$

This infinite factor should be canceled by normalization and will not be mentioned later. Note that the formalism contains one sources $\mathbf{j}_a$ conjugate to the coordinates $\mathbf{u}_a$ only, see (101) and (93).

So, described mapping gives the measure:

$$DM_j(\xi,\eta,\lambda,Q)=\prod_{x,t;a} d\lambda_a dq_a d\xi d\eta\delta\left(\dot{\lambda}_a\right)\delta\left(\dot{q}_a+\frac{\delta h_j}{\delta\lambda_a}\right)$$

$$\delta\left(\dot{\xi}-\frac{\partial h_j}{\partial\eta}\right)\delta\left(\dot{\eta}+\frac{\partial h_j}{\partial\xi}\right) \tag{103}$$

We have took into account here that $(u,p)_a$ are q_a independent. The Hamiltonian h_j is defined by eq.(101):

$$2gh_j=\int d^3x\left(p_a^2+\mathbf{B}_a^2(u)\right)+\int d^3x\mathbf{j}_a\mathbf{u}_a\equiv h+J. \tag{104}$$

where h is the unperturbated by $\mathbf{j}_a$ Hamiltonian.

Helping the proposition V, we can exclude the q_a dependence:

$$DM_j(\xi,\eta,\lambda)=dR\prod_{x;a} d\lambda_a d\xi d\eta\delta(\dot{\lambda}_a)\delta\left(\dot{\xi}-\omega-j_\xi\right)\delta\left(\dot{\eta}-j_\eta\right) \tag{105}$$

where the 'velocity' $\omega = \partial h/\partial \eta$. The perturbations generating operator takes the form:

$$2\mathcal{K}(je) = \int dt \{\hat{j}_\xi \hat{e}_\xi + \hat{j}_\eta \hat{e}_\eta\}. \tag{106}$$

At the same time one should replace in (85) $\mathbf{e}_a$ on

$$\mathbf{e}_a(x) = e_\xi(t) \frac{\partial \mathbf{u}_a(x; \xi, \eta, \lambda)}{\partial \eta(t)} - e_\eta(t) \frac{\partial \mathbf{u}_a(x; \xi, \eta, \lambda)}{\partial \xi(t)}. \tag{107}$$

As follows from (105) we should consider the time independent gauge transformations:

$$\dot{\lambda}_a(x) = 0. \tag{108}$$

To remove this constraint we should generalize equation (100). So, if we consider the equation:

$$\{\mathbf{u}_a(x; \xi, \eta, \lambda), h_j\} = \frac{\delta H_j}{\delta \mathbf{p}_a(x)} - \Omega_a(x) \frac{\partial \mathbf{u}(x; \xi, \eta, \lambda)}{\partial \lambda_a} \tag{109}$$

instead of first equation in (100) then one should replace in (105)

$$\prod_{x;a} d\lambda_a(x) \delta(\dot{\lambda}_a(x)) \to \prod_{x;a} d\lambda_a(x) \delta(\dot{\lambda}_a(x) - \Omega_a(x)), \tag{110}$$

where $\Omega_a(x)$ is the arbitrary function of y and t. This is the mostly general representation for gauge measure in our formalism.

In result, the main elements of quantum Yang-Mills theory in the $\mathcal{G}/\mathcal{H}$ space looks as follows:

(i) The measure

$$DM_j(\xi, \eta, \lambda) = dR \prod_{x;a} d\lambda_a d\xi d\eta \delta(\dot{\lambda}_a(x) - \Omega_a(x)) \delta(\dot{\xi} - \omega - j_\xi) \delta(\dot{\eta} - j_\eta) \tag{111}$$

Using the definition (25), one may note that

$$\int \prod_{x;a} d\lambda_a \delta(\dot{\lambda}_a(x) - \Omega_a(x))$$

means integration over all functions $\lambda_a(y, t)$ of the arbitrary given time dependence. At the same time

$$\frac{\int \prod_{x;a} d\lambda_a \delta(\dot{\lambda}_a(x) - \Omega_a(x))}{\int \prod_{x;a} d\lambda_a} \equiv 0. \tag{112}$$

Therefore our normalization on the gauge group volume differs from ordinary one. But this will not affect the result since all contributions will be gauge invariant.

(ii) The quantum perturbations generating operator

$$2\hat{K}(\mathbf{je}) = \int dt \{\hat{\mathbf{j}}_\xi \cdot \hat{\mathbf{e}}_\xi + \hat{\mathbf{j}}_\eta \cdot \hat{\mathbf{e}}_\eta\} \tag{113}$$

(iii) The interactions functional $U(\mathbf{u}, \bar{e})$ depends on

$$\mathbf{e}_a = \mathbf{e}_\xi \cdot \frac{\partial \mathbf{u}_a}{\partial \eta} - \mathbf{e}_\eta \cdot \frac{\partial \mathbf{u}_a}{\partial \xi}. \tag{114}$$

Note the motion along λ orbits is exactly classical and the dependence of nondynamical variables was disappeared.

3.4 Gauge invariance

We wish to quantize the theory without gauge fixing *ansatz* and, therefore, the theory contains three *independent* potential $\mathbf{u_{ia}}$, $i = 1, 2, 3$ for each color index a. We may avoid this problem with the unphysical degrees of freedom if the theory would depend only from the gauge-invariant observable quantities: the color electric, $\mathbf{E}_a$, and magnetic, $\mathbf{B}_a$, fields.

Proposition VI. *Each order over* $1/g$ *is explicitly gauge invariant*

The interactions functional U has following explicit form:

$$-3!U(\mathbf{u}, e) = \frac{1}{g} \int dx \prod_{k=1}^{3} \left\{ e_{a_k} \frac{\partial}{\partial u_{a_k}} \right\} F^{\mu\nu a} F_{\mu\nu a},$$

where e_a was defined in (114). Using this definition, we find:

$$-3!U(\mathbf{u}, e) = \int dx \prod_{k=1}^{3} \left\{ \left[\mathbf{e}_\xi \cdot \frac{\partial \mathbf{u}_a}{\partial \eta} - \mathbf{e}_\eta \cdot \frac{\partial \mathbf{u}_a}{\partial \xi} \right] \frac{\partial}{\partial \mathbf{u_{a_k}}} \right\} F^{\mu\nu a} F_{\mu\nu a}. \tag{115}$$

The summation over repeated indices is assumed.

Last expression is manifestly gauge invariant since the operator is singlet of gauge transformations and $F^{\mu\nu a} F_{\mu\nu a}$ is the gauge invariant quantity.

3.5 Divergences

The expression (115) may be written in the form:

$$-3!U(\mathbf{u}, \bar{e}) = \int dt \prod_{k=1}^{3} \left\{ \left[\mathbf{e}_\xi \cdot \frac{\partial \mathbf{u}_a}{\partial \eta} - \mathbf{e}_\eta \cdot \frac{\partial \mathbf{u}_a}{\partial \xi} \right] \frac{\partial}{\partial \mathbf{u_{a_k}}} \right\} \mathcal{L}(u), \tag{116}$$

where

$$\mathcal{L}(u) = \int d^3x F^{\mu\nu a} F_{\mu\nu a}$$

is the Yang-Mills Lagrangian.

Result of action of the perturbation generating operator gives the expression:

$$\mathcal{N}(u) = \int DM(\xi, \eta) : e^{-2iU(u,e)} :, \tag{117}$$

where the operator

$$-3!(2i)^3 \mathcal{U}(u, e) = \int dt \prod_{k=1}^{3} \left\{ \left[\frac{\delta}{\delta \mathbf{j}_\xi} \cdot \frac{\partial \mathbf{u}_a}{\partial \eta} - \frac{\delta}{\delta \mathbf{j}_\eta} \cdot \frac{\partial \mathbf{u}_a}{\partial \xi} \right] \frac{\partial}{\partial \mathbf{u_{a_k}}} \right\} \mathcal{L}(u), \tag{118}$$

where $u_(ia)$ depends on the solution of equations:

$$\dot{\xi} - \omega(\eta) = j_\xi, \quad \dot{\eta} = j_\eta \tag{119}$$

and the measure is j_X, $X = \xi, \eta$ independent:

$$DM = dR \prod_a \prod_{y,t} D\lambda_a \delta(\dot{\lambda}_a - \Omega) \delta(\dot{\xi} - \omega(\eta)) \delta(\dot{\eta}).$$

Such 'shift' is possible since the equations (119) are linear over j_X.

We can conclude that if $u_{a\mu}$ is not singular,

$$|S(u)| < \infty, \tag{120}$$

then the theory did not contain divergences since the differential operator in (118) can not change convergence of the time integrals. Notice that the $O(4) \times O(2)$ solution obey this property [3].

4 Conclusion

It was shown that exist such formulation of the quantum Yang-Mills theory which is (a) divergences free (at least in the vector fields sector), (b) did not contain the gauge ghosts and (c) is sufficiently consistent, i.e. the quantization scheme is free from the Gribov ambiguities.

It was shown in [2] that if $\partial(\mathcal{G}/\mathcal{H})$ is the boundary then the quantum corrections are accumulated on this boundary, i.e. the intersection $\partial u_{a\mu} \bigcap \partial(\mathcal{G}/\mathcal{H})$, where $\partial u_{a\mu}$ is the flow in the $\mathcal{G}/\mathcal{H}$ coordinate system, defines the value of quantum corrections. If $\partial u_{a\mu} \bigcap \partial(\mathcal{G}/\mathcal{H}) = 0$ then the semiclassical approximation is exact. This is the crucial property of our topological QCD.

For this reason the tQCD seems attractive and the question, may it take the place of pQCD is seems important. The experimentally examined consequences of the tQCD would be extremely interesting and they will be investigated in the first place.

Being convergent, the exactness of estimation of the measurables in tQCD should be higher then in the 'logarithmic' pQCD. Moreover, the convergence means that the main contributions are accumulated on the large distances. This property is typical for hadron physics. Therefore, the main point of our future publications would be the prediction of the small-scale effects, where we can compare our approach with pQCD.

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A Appendix. Reduction of the space degrees of freedom

Action of the operator $\exp\{-i\mathcal{K}\}$ leads to the expression:

$$\mathcal{N}(u) = \int DM_j(\xi, \eta) : e^{-2i\mathcal{U}(u,j)} : . \tag{a.1}$$

where

$$-3!(2i)^3\mathcal{U}(u,j) = \int d^3x dt \left\{ \left[\frac{\delta}{\delta \mathbf{j}_\xi} \cdot \frac{\partial \mathbf{u}_a}{\partial \eta} - \frac{\delta}{\delta \mathbf{j}_\eta} \cdot \frac{\partial \mathbf{u}_a}{\partial \xi} \right] \frac{\partial}{\partial \mathbf{u}_{\mathbf{a_k}}} \right\} \mathcal{L}(u) \tag{a.2}$$

and the colons in (a.1) mean the 'normal product', when the variational derivatives over j_X in the expansion of $\exp\{-2i\mathcal{U}(u,j)\}$ stay to the left of all functions.

The measure

$$DM_j(\xi,\eta)=\prod_{y,t}d\xi d\eta\delta(\dot{\xi}-\omega_\eta-j_\xi)\delta(\dot{\eta}+\omega_\xi-j_\eta)$$

Then, to calculate the remaining integral in (a.1), one should find solution of inhomogeneous equations:

$$\dot{\xi}(y,t)-\omega_\eta(y,t;\xi,\eta)=j_\xi(y,t),\quad \dot{\eta}(y,t)+\omega_\xi(y,t;\xi,\eta)=j_\eta(y,t), \tag{a.3}$$

where

$$\omega_X(y,t;\xi,\eta)=\delta h(\xi,\eta)/\delta X(y,t).$$

As follows from (a.2), if some operators $\hat{j}_{X'}$ over the 'auxiliary' variable X' did not contained in $\mathcal{U}(u,j)$ then the auxiliary variables X' should obey the homogeneous, classical, equations, with $j_{X'}=0$ in the right hand side.

The solutions of inhomogeneous equation (a.3) will be searched expanding over j_X:

$$\begin{aligned}\xi(y,t)=\xi^0(y,t)+\int d^4x'\xi^1_\xi(y,t;y',t')j_\xi(y',t')+\\ +\int d^4x'\xi^1_\eta(y,t;y',t')j_\eta(y',t')+...,\\ \eta(y,t)=\eta^0(y,t)+\int d^4x'\eta^1_\eta(y,t;y',t')j_\eta(y',t')+\\ +\int d^4x'\eta^1_\xi(y,t;y',t')j_\xi(y',t')+...\end{aligned} \tag{a.4}$$

So, the equations:

$$\dot{\xi}^0(y,t)=\omega_\eta(y,t;\xi^0,\eta^0),\ \ \dot{\eta}^0(y,t)=-\omega_\xi(y,t;\xi^0,\eta^0) \tag{a.5}$$

should be solved in the lowest order over j_X. The function $u(y;\xi(y,t),\eta(y,t))$ should obey the 'boundary' property:

$$u(y;\xi(y,t),\eta(y,t))|_{j=0}=u(y;\xi^0,\eta^0)=\mathrm{u}(y,t;\xi_0,\eta_0) \tag{a.6}$$

where ξ_0 and η_0 are the integration constants of the Lagrange equation (11). The equality (a.6) defines the starting set of the necessary variables ξ and η. Notice that, as follows from proposition **III**, the quantum perturbations should not change this set.

Let us distinguish the variables $\xi\in\mathcal{G}/\mathcal{H}$ by the equality:

$$\left.\frac{\delta}{\delta\xi}h\right|_{j_X=0}=0. \tag{a.7}$$

This assumes that the set η can be expressed through the set conserved generators. In the example (1), they are the generators of translation and special conformal transformation. Notice, that the proposition **III** mens that the quantum perturbations did not alter this definition.

Inserting (a.7) into (a.5) we find at $j_X=0$ the equations:

$$\dot{\xi}^0(y,t)=\omega_\eta(\eta^0)\equiv\omega(\eta^0),\ \ \dot{\eta}^0(y,t)=0. \tag{a.8}$$

The functions with arbitrary y dependence may satisfy this equations. Using solution of this equations:

$$\xi^0(y,t) = \omega(\eta^0)t + \xi_0, \quad \eta^0(y,t) = \eta_0, \tag{a.9}$$

where ξ_0 and η_0 are the integration constants, we will see that the dependence on y in (a.6) did not play any role because of the degeneracy over y. For this reason we will put out the y dependence in ξ^0 and η^0.

It is not hard to show that the degeneracy over y will conserved in arbitrary order over j_X. Indeed, inserting the expansions (a.4) into the equations (a.3), we find in the first order over j_ξ:

$$\partial_t \xi^1_\xi(y,t;y',t') - \xi^1_\xi(y,t;y',t') \left.\frac{\delta^2 h(\xi,\eta)}{\delta\xi(y',t')\delta\xi(y,t)}\right|_{j=0} -$$
$$-\xi^1_\eta(y,t;y',t') \left.\frac{\delta^2 h(\xi,\eta)}{\delta\eta(y',t')\delta\xi(y,t)}\right|_{j=0} = \delta(y-y')\delta(t-t').$$

Notice that

$$\left.\frac{\delta h(\xi,\eta)}{\delta\xi(y,t)}\right|_{j=0} = \frac{\delta}{\delta\xi(y,t)}\left\{ h(\xi,\eta)|_{j=0}\right\} = 0,$$

where (a.7) was used. Therefore, the equation for ξ^1_ξ have a structure:

$$\dot{\xi}^1_\xi(y,t;y',t') = \delta(y-y')\delta(t-t'), \tag{a.10}$$

where the boundary conditions (a.9) was applied. Notice that this equation is linear.

Inserting the solution of equation (a.10):

$$\xi^1_\xi(y,t;y',t') = \delta(y-y')g(t-t'), \tag{a.11}$$

where $g(t-t')$ is the Green function defined in [2], into (a.4), we find the term

$$\sim \int dt' g(t-t') j_\xi(y,t').$$

So, the y dependence is contained in the auxiliary source j_ξ only. For this reason it can not play dynamical role. The same phenomena one can observe considering other terms in the decomposition (a.4).

Therefore, admitting that the quantum perturbations switched on adiabatically, i.e. may be taken into account perturbatively, and for this reason are unable to change the topology of the classical trajectory $u(y;\xi,\eta)$, the proposition **III**, one may conclude that it is enough to take $\xi = \xi(t)$ and $\eta = \eta(t)$ in the considered scalar theory.

References

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